Heavy Ion Collisions and the Density Dependence of the Local Mean Field[1]

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Abstract:

We study the effect of the density dependence of the scalar and the vector part of the nucleonic self-energy in Relativistic Quantum Molecular Dynamics (RQMD) on observables like the transversal flow and the rapidity distribution. The stability of nuclei in RQMD is greatly improved if the density dependence is included in the self-energies compared to a calculation assuming always saturation density of nuclear matter. Different approaches are studied: The main results are calculated with self-energies extracted from a Dirac-Brückner-Hartree-Fock G-matrix of a one boson exchange model, i.e. the Bonn potential. These results are compared with those obtained by a generalization of static Skyrme force, with calculations in the simple linear Walecka model and results of the Brückner-Hartree-Fock G-matrix of the Reid soft core potential. The transversal flow is very sensitive to these different approaches. A comparison with the data is given.

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1. Introduction

The dynamics of heavy-ion reactions depends on the mean field and the description of binary collisions. Hence, the mean field and the cross sections for binary collisions are the main ingredients of transport models like Quantum Molecular Dynamics (QMD) [1, 2] and the BUU-type models [3, 4, 5, 6, 7, 8] which are proven to describe successfully the dynamics of heavy-ion collisions. Finally, both ingredients should be determined from the same parameter free microscopic force. The strong repulsive short range correlations of these forces necessitate a non-pertubative self-consistent treatment as done in Brückner theory.

In the non-relativistic regime such a self-consistent description of heavy-ion reactions was done in the framework of QMD[9, 10]. In this approach self-consistent potentials as well as in medium cross sections are extracted from non-relativistic G-matrix elements which are given by Brückner-Hartree-Fock (BHF) calculations. In these calculations the Bethe-Goldstone equation including an angle averaged Pauli operator was solved for the situation of two pieces of nuclear matter penetrating each other. As a bare nucleon-nucleon interaction the Reid-soft-core potential is used in this approach.

In the covariant extension of QMD, i.e. Relativistic QMD (RQMD) [11, 12, 13], up to now only static Skyrme forces combined with the so-called Cugnon parametrization of the nucleon-nucleon cross section were used. These generalized Skyrme forces are treated as scalar potentials in the framework of Constrained Hamilton Dynamics. But the full Lorentz-structure of the nucleon-nucleon interaction contains large scalar and vector components. Effects of these relativistic forces were already studied in the framework of covariant generalizations of models of the BUU-type [14, 15, 16], mostly in the framework of the $\sigma - \omega$ model, but not yet in RQMD.

In order to determine the scalar and vector components of the self-energy of the nucleon from realistic forces in a covariant fashion the solution of the Bethe- Salpeter equation with Pauli-operator is needed, which can be achieved in a three dimensional reduction of the Bethe-Salpeter equation as given by the Thompson equation. Nuclear matter calculations [17, 18] in this framework are done by using one boson exchange potentials (OBEP), e.g. the Bonn potential, as the bare nucleon-nucleon interaction. The self-energy components extracted from these Dirac-Brückner-Hartree-Fock (DBHF) calculations show a strong density dependence while the momentum dependence is moderate. Therefore, in the present approach we like to study as a first step the density dependence of the self-energy components in a local density approximation (LDA). For this purpose we use in RQMD simulations self-energy components as extracted from G-matrix elements determined by DBHF calculations using the Bonn potential as well as a parametrization [19] footing on BHF calculations using the Reid-soft-core potential. In both approaches we determine observables which reflect on the time evolution of the phase space, namely, the rapidity distribution and the transversal flow inside the reaction plane.

In the second chapter we sketch our RQMD approach. In addition, in this chapter we give shortly an outline how the results extracted from the Brückner approach for the mean field using the solutions of the non-relativistic Bethe-Goldstone equation and the relativistic Bethe-Salpeter equation as well are used within RQMD in a local density approximation. In the third chapter we discuss the results and compare them with different approaches and data. In the last chapter we finally summarize our main conclusion.

2. Density Dependent Interactions within RQMD

In this section we give a brief description of the formalism of relativistic Quantum Molecular Dynamics (RQMD) with relativistic self-energies.

In the formalism of Constrained Hamiltonian Dynamics [20, 21], the original 8N dimensional phase space of N interacting relativistic particles is reduced to usual 6N dimensions by 2N constraints. These constraints fix the individual energies and parametrize the world lines by fixing the relative time coordinates.

The energies are fixed by N on-shell conditions

$$K_i = (p_i^\mu - \Sigma_i^\mu)(p_{i\mu} - \Sigma_{i\mu}) - (m_i - \Sigma_i^{(s)})^2 = 0, \tag{1}$$

which contain scalar and vector potentials. Hence, the constraints (1) take the full Lorentz structure of the NN interaction into account explicitly. We regard the self-energy components $\Sigma_i^{(s)}$ and Σ_i^μ as sums of two-body mutual interactions-at-a-distance, i.e. $\Sigma_i^{(s)} = \sum_{j\neq i} \Sigma_{ij}^{(s)}$ and $\Sigma_i^\mu = \sum_{j\neq i} \Sigma_{ij}^\mu$. In order to use the self-energies provided by relativistic NN interactions, we adopt a kind of local density approximation (LDA). In the relativistic Hartree approximation the self-energies are proportional to the scalar density ρ_s and the baryon current B^μ, i.e.

$$\Sigma^{(s)} = \Gamma_s \rho^s, \ \Sigma^\mu = \Gamma_v B^\mu. \tag{2}$$

Interpreting the sum of the Lorentz scalar two-body interaction densities

$$\rho_{ij} = \frac{1}{(4\pi L)^{3/2}} exp(q_{Tij}^2/4L), \tag{3}$$

as a scalar density $\rho_i^s = \sum_{j\neq i} \rho_{ij}$ one can define in addition the baryon current in a consistent way and gets the self-energies of a particle i as

$$\Sigma_i^{(s)} = \Gamma_s \sum_{j\neq i} \rho_{ij}, \ \Sigma_i^\mu = \Gamma_v \sum_{j\neq i} \rho_{ij} u_j^\mu. \tag{4}$$

Here $q_{Tij}^\mu = (g^{\mu\nu} - \hat{p}_{ij}^\mu \hat{p}_{ij}^\nu) q_{ij\nu}$, $q_{ij}^\mu = q_i^\mu - q_j^\mu$, $p_{ij}^\mu = p_i^\mu + p_j^\mu$, $\hat{p}_{ij} = p_{ij}/(p_{ij}^2)^{1/2}$ and u_j^μ being the 4-velocity of particle j. In this sense the self-energies are given by sums of

two-body mutual interactions-at-a-distance. The dependence of these self-energies on q_{Tij}^2 ensures the multi-time description used in RQMD without further additional approximations. A detailed discussion proving the reliability of these definitions is given in the appendix.

In the case of realistic NN interactions the strong short-range components require to take account for the effects of two-body correlations. This can be attained in the framework of Brückner theory. Results of self-consistent Dirac-Brückner calculations can be used in the approach discussed above with help of effective coupling functions Γ_s and Γ_v, which are density dependent. This density dependence is usually not included. We shall show that the inclusion of this density dependence improves the description greatly. In this way, realistic forces extracted from Dirac-Brückner calculations can be implemented in RQMD.

For the actual simulations we have used 4 kinds of self-energies extracted from the following models and theories:

(1) Skyrme model for the soft equation of state [12, 13] (Skyrme),

(2) σ-ω model [22, 23] (QHDI),

(3) Brückner Hartree Fock theory with Reid soft core potential [19] (BHF),

(4) Dirac-Brückner Hartree Fock theory with Bonn potential [24] (DBHF).

In order to obtain fully covariant constraints, we have fitted the density dependent coupling constants as functions of the local scalar density to reproduce the original calculations of scalar and vector self-energies of models (3) and (4) by means of the Lagrange interpolation method. The quality of the fit is shown in Fig.1.

We have followed the ref.[13] for the choice of the other additional N constraints to specify the world lines of the particles:

$$\chi_i = \sum_{j \neq i} w_{ij} p_{ij}^{\mu} q_{\mu ij} = 0, \ i = 1, ..., N-1 \tag{5}$$

$$\chi_N = \hat{P}^{\mu} Q_{\mu} - \tau = 0 \tag{6}$$

with $\hat{P}^\mu = P^\mu/(P^2)^{1/2}$, $P^\mu = \sum_i p_i^\mu$, $Q^\mu = \sum_i q_i^\mu/N$ and the dimensionless scalar weight function

$$w_{ij} = \frac{1}{q_{ij}^2/L_c} exp(q_{ij}^2/L_c) \tag{7}$$

with $L_c = 8.66$ fm^2 and τ denotes a global time evolution parameter.

The total Hamiltonian is now expressed as

$$H = \sum_{i=1}^{2N-1} \lambda_i \phi_i \tag{8}$$

where $\phi_i = K_i$ for $i \leq N$ and $\phi_i = \chi_{i-N}$ for $N+1 \leq i \leq 2N-1$, then the equations of motions are given by

$$dq_i^\mu/d\tau = [H, q_i^\mu], \;\; dp_i^\mu/d\tau = [H, p_i^\mu] \tag{9}$$

where $[A, B]$ means the Poisson bracket of phase space functions A and B. The evolution of the system can be computed by integrating the set of above equations (9), but we have to determine the unknown Lagrange multipliers $\lambda_i(\tau)$ to calculate the dynamics. This can be done using the fact that the complete set of $2N$ constraints ϕ_i ($\phi_{2N} = \chi_N$) must be fulfilled during the whole time evolution, i.e.

$$d\phi_i/d\tau = \partial\phi_i/\partial\tau + [H, \phi_i] = 0. \tag{10}$$

Using the Hamiltonian (8), one gets for the Poisson bracket

$$[H, \phi_i] = \sum_{j=1}^{2N-1} \lambda_j [\phi_j, \phi_i] \tag{11}$$

thus inverting the matrix $C_{ji} = [\phi_j, \phi_i]$, we obtain the multipliers as

$$\lambda_i = C_{iN}^{-1}. \tag{12}$$

If the Dirac's first class condition is fulfilled, i.e. $[K_i, K_j] = 0$, (in our case the condition has been confirmed numerically during the whole time evolution.) the Hamiltonian is reduced into a simpler form:

$$H = \sum_{i=1}^{N} \lambda_i K_i \tag{13}$$

$$\lambda_i = \Delta_{iN}^{-1} \tag{14}$$

where $\Delta_{ji} = [K_j, \chi_i]$ is a submatrix of the constraint matrix C_{ji}.

For more details, especially for the foundations of these formalisms, we refer to [1] in case of QMD and to [11] and [13] in case of RQMD. We note that the binary collisions are dealt in RQMD in the same way as in the non-covariant QMD by using Monte Carlo methods.

3. Results

First we compare the stability of RQMD nuclei described with different interactions obtained from the above mentioned 4 models and theories. It is important to check the stability at actual boosting energies, since the momentum dependence of the self-enegies hides the trivial analytic expressions of covariance. We adopt the energy at 1.8 GeV/A for later simulations. Figure 2 shows the root mean square radii of Ca versus time, with 6 different approaches: Figure 2 contains the time dependence of phenomenological models, i.e. static Skyrme and linear Walecka models. As is easily seen, although the Skyrme model works well, but in the simple linear Walecka model the nuclei dissassemble after several time steps. This is understandable: the strong constant couplings directly reflect on strong repulsive components of the force when the local density reaches high values because of the Fermi motion of the nucleons, however it might be expected that the density dependence of the self-energies recovers the discrepancy, since the self-energies are softened in high density domain if the two body correlations are included in Brückner theory (see Fig.1), thus the correlation, i.e. density dependence of the couplings would give a counter balance automatically inside of nuclei. In order to visualize this mechanism clearly, thus we plot the time dependence with microscopic density dependent self-energies and the self-energies at the saturation point from BHF and DBHF calculations in Figs.2. One can see that the density dependence of the self-energies strongly improved the stability of the nuclei.

One of the most interesting physical observable in heavy ion collision at intermediate energies are rapidity distributions and the collective transverse flow. These quantities are very sensitive to the interactions used in the models, thus frequently used to investigate the nuclear equation of state far from the saturation point. In order to be able to compare with the experimental data, we have performed simulations of the nearly symmetric collision Ar+KCl at 1.8 GeV/A which were measured with the streamer chamber at the BEVALAC at LBL [25]. In the calculation we are using the above 3 kinds of stable nuclei, i.e. simulated by (1) Skyrme (2) BHF (3) DBHF self-energies and compared with the experimental data [26]. The initialization of the target and the projectile has been carried out by standard Monte Carlo methods as described in ref.[1]. According to the ref.[26], we have chosen the impact parameter range to be $0 \leq b \leq 2.3$ fm. The standard Cugnon parametrization is used for the binary nucleon-nucleon cross sections [27].

The collective transverse flow obtained from our simulation is plotted in Fig.3. It is quite remarkable that the conventional Skyrme force (soft EOS) is not able to reach the experimentally measured magnitude of the flow distribution at this energy. The reason is clear: The force contains only static interactions between constituent particles and no explicit energy dependence. However, on the contrary, the experimental data lie in between DBHF and BHF results. The Schrödinger equivalent potential extracted from the DBHF self-energies shows a linear energy dependence in our approach. Hence one gets a repulsive interaction at high relative momenta which produce a strong sidewards flow, which is reflected on the P_x/A. There is still deviation between the theory and the experiment, it might be overcome if one takes the configuration dependence (deviation of the momentum distribution from a Fermi sphere) in the momentum space of the selfenergies into account, means by using local configurations which can be parametrized by two penetrating fermi spheres [19].

The rapidity distributions is also shown in Fig.4. The asymmetric property of the distribution is due to the system we study. One can notice that the Skyrme result exhibits less and BHF gives the highest stopping, but the differences between the various approaches are rather small. We found that the Skyrme provides a little bit less Pauli blocking (around 10 percent compared with the realistic forces.). And also, the stability of single nuclei might affect on this quantity, since it is not quite stable using BHF in comparison when using static Skyrme interactions or interactions extracted from DBHF (see Fig.2).

4. Conclusions

We investigated the effect of realistic forces in comparison to phenomenological forces on the sidewards flow and the rapidity distribution created during heavy-ion reactions by using the covariant RQMD approach. The realistic forces are extracted from Dirac-Brückner calculations and non-relativistic Brückner calculations as well while the phenomenological forces are determined by local static Skyrme interactions or in the framework of a simple linear Walecka model.

It was proven that single nuclei are stable during the time span needed for a simulation of heavy-ion collisions if the full density dependence (beyond a linear one) of the interaction is included when using realistic forces.

The rapidity distribution at the final state shows that realistic interactions create slightly more stopping than the static Skyrme interaction.

Due to the energy dependence and the correct density dependence of these realistic forces the flow is enhanced in comparison to calculations using static Skyrme interactions. Furthermore, the results obtained using realistic forces are much closer to the data. The agreement with the data might be improved if the self-energies are determined including the full local momentum distributions (going beyond a Fermi sphere) as it will be done in the near future.

Appendix The usual density used in a non-relativistic framework is defined as a sum over N Gaussians describing the N nucleons [1], i.e.

$$\rho_G(\vec{r},t) = \frac{1}{(2\pi L)^{3/2}} \sum_j^N \mathrm{e}^{-(\vec{r}-\vec{r}_j(t))^2/2L}. \tag{15}$$

$L = 1.08$ fm^2 fixes the width of a Gaussian. Reasonable values for ρ_G within the multi-time description of RQMD are achieved by propagating all nucleons (forward or backward) to the same time coordinate in the frame in which ρ_G has to be determined, e.g. in the CMS.

In order to prove the reliability of the definitions for the scalar density ρ_S and the baryon density ρ_B used in eqs.(4) we determined the density profile of ρ_G, ρ_S and ρ_B of a Ca nucleus in its rest frame and for a boosted situation (boosted to the CMS energy of a symmetric collision at 2 GeV/A) as well. The results of these test calculations are shown in Fig. 5. In the rest frame of the nucleus ρ_G and ρ_B coincide as it should be while ρ_S is smaller in the central region. In the boosted situation ρ_G and ρ_B are enhanced by a $\gamma-$factor compared to ρ_S. ρ_G and ρ_B show a quite similar shape in the boosted situation which justifies that the definition used to determine ρ_B in RQMD simulations is reliable. ρ_G shows not the full Lorentz contraction because its determination in RQMD simulations contains approximations as mentioned above. The reader should note that the density ρ_G given at a special point $\vec{r}$ has half the width of the so-called interaction density evaluated at the center of a Gaussian $\vec{r}_i$ of a particle i. Hence, ρ_B and ρ_S as well are also determined by using half the width of the corresponding interaction density when evaluated at a special point in space time as done in the calculations shown in Fig.5.

References

[1] J. Aichelin, Phys.Rep. **202**, 233 (1991).

[2] J. Aichelin and H. Stöcker, Phys.Lett. **B176**, 14 (1986).

[3] H. Kruse, B. Jacak and H. Stöcker, Phys.Rev.Lett. **54**, 289 (1985).

[4] A. Bonasera, M. Di Toro and H.H. Wolter, Proc.XXVII Winter Meeting, Bormio 1989, eds. I. Iori et al..

[5] C. Grégoire, B. Remaud, F. Sébille, L. Vinet and Y. Raffray, Nucl.Phys. **A465**, 317 (1987).

[6] W. Bauer, G.F. Bertsch, W. Cassing and U. Mosel, Phys.Rev. **C34**, 2127 (1986).

[7] G.F. Bertsch and S. Das Gupta, Phys.Rep. **160**, 189 (1988).

[8] G.F. Bertsch, H. Kruse and S. Das Gupta, Phys.Rev. **C29**, 673 (1984).

[9] J. Jaenicke, J. Aichelin, N. Ohtsuka, R. Lineden and A. Faessler, Nucl.Phys. **A536** (1992) 102.

[10] D.T. Khoa, N. Ohtsuka, M.A. Matin, A. Faessler, S.W. Huang, E. Lehmann and R.K. Puri, Nucl. Phys. **A548 (1992)** 102.

[11] H. Sorge, H. Stöcker and W. Greiner, Ann.Phys. **192**, 266 (1989).

[12] T. Maruyama, S.W. Huang, N. Ohtsuka, G.Q. Li, A. Faessler and J. Aichelin, Nucl.Phys. **A534**, 720 (1991).

[13] E. Lehmann, R.K. Puri, A. Faessler, G. Batko and S.W. Huang, Phys.Rev. **C51** (1995) 2113.

[14] C.M. Ko, Q. Li and R. Wang, Phys.Rev.Lett. **59**, 1084 (1987).

[15] W. Cassing and U. Mosel, Prog.Part.Nucl.Phys. **25**, 235 (1990).

[16] C. Fuchs, L. Sehn, H.H. Wolter, Nucl.Phys. **A545**, 151c (1992).

[17] R. Brockmann and R. Machleidt, Phys.Lett. **B149** (1984) 283.

[18] B. ter Haar and R. Malfliet, Phys.Reports **149** (1987) 207.

[19] H. Elsenhans, L. Sehn, A. Faessler, H. Müther, N. Ohtsuka and H.H. Wolter, Nucl.Phys. **A536** (1992) 750.

[20] P.A.M. Dirac, Rev.Mod.Phys. **21**, 392 (1949).

[21] J. Samuel, Phys.Rev. **D26**, 3475, 3482 (1982).

[22] J.D. Walecka, Ann.Phys. **83** (1974) 491.

[23] B.D. Serot and J.D. Walecka, Adv.Nucl.Phys. **16** (1988) 1.

[24] H.F.Boersma and R. Malfliet, Phys.Rev. **C49** (1994) 233.

[25] H. Ströbele et al., Phys.Rev. **C27** (1983) 1349.

[26] P. Danielewicz and G. Odyniec, Phys.Lett. **B157** (1985) 141.

[27] J. Cugnon and M.-C. Lemaire, Nucl.Phys. **A489**, 781 (1988).

Figure Captions

Fig. 1: Effective Couplings as functions of the scalar density. Solid lines asign DBHF(Dirac Brueckner Hartree Fock)/BHF(Brueckner Hartree Fock) calculations, dashed lines show the fit used in RQMD simulations. QHDI denotes the values from the $\sigma - \omega$ model at the saturation point.

Fig. 2: Root mean square radii of single Ca nuclei boosted to the CMS energy of a symmetric collision at 1.8 GeV/A as a function of time. The different approaches DBHF, BHF and QHDI are explained in the caption of figure 1. 'sat.' indicates for $\Gamma_s(\rho_s)$ and $\Gamma_v(\rho_s)$ at the saturation density.

Fig. 3: Flow distribution versus rapidity at the final state (60fm/c) of Ar+KCl collision at 1.8 GeV/A and $b \leq 2.3$ fm. The data are taken from [26].

Fig. 4: Rapidity distribution at the final state (60 fm/c) of Ar+KCl collisions at 1.8 GeV/A and $b \leq 2.3$ fm.

Fig. 5: Usual density ρ_G, scalar density ρ_S and baryon density ρ_B (definitions see text) for a single Ca nucleus in the rest frame of the nucleus (left part) and in the CMS (right part).